ELUCIDATING THE ROLE OF THE SHAPE OF THE PSD IN PRECIPITATION ESTIMATION

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interests in mp

• users of mp parameterisations

• main interest in the climatology: how those mps operate in different regions and regimes

• role of the DSD assumptions

• also interested in the assumptions behind the parameterizations

• test / verify / validate
Direct Interactions of Parameterizations

- MP
- Cloud detrainment
- Non convective rain
- Convective rain
- Radiation
- Surface emission/ albedo
- Downward SW, LW
- Surface SH, LH
- PBL
- T, Qv, wind
Multi-Physics (MPP)

Error in the Storm Center Location Compared with Ground Radar

- **Kessler**
- **Lin**
- **WSM-3**
- **WSM-5**
- **WSM-6**
- **Goddard**
- **Thompson**

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*Cumulus param.:*
- **B-M-J**
- **G-D**
mp types

- bin microphysics

- bulk microphysics
Particle Size Distribution

\[ p(D) = D^\mu \Lambda^{\mu+1} \frac{e^{-\Lambda D}}{\Gamma(\mu + 1)} \]

- \( p(D) \in [0, 1] \)
- \( p(\Omega) = 1 \)

\[ p(D_1 \cup D_2 \cup \ldots \cup D_n) = \sum_{i=1}^{n} p(D_i) \]

\[ n(D) = N_T p(D) = N_T D^\mu \Lambda^{\mu+1} \frac{e^{-\Lambda D}}{\Gamma(\mu + 1)} \]
Issue #1

• One-mode parameterizations have been improved
• Two and three modes params are becoming more common
• Computer resources allow for more complex models
• Why some people claim that simple parameterisations work the best?
Issue #2

- Empirical fits to for instance the gamma function are not proper PDFs

\[ n(D)_{Tes} = N_0^* F_\mu(D/D_m) \]

\[ F_\mu(X) = \frac{\Gamma(4)}{4^4} \frac{(4 + \mu)^{4+\mu}}{\Gamma(4 + \mu)} X^\mu e^{-(4+\mu)X} \]
What is a PDF?

\[ p(D) \in [0, 1] \]
\[ \int p(D) dD = 1 \]
\[ p(D) \in \mathbb{R}, p(D) \geq 0 \]
\[ p(D_1 \cup D_2 \cup \ldots D_n) = \sum_{i=1}^{n} p(D_i) \]
\[ n(D) = N_T \cdot p(D) \]
Why do we want DSDs to be probabilistic ($N_T$-linked with PDFs)?

- Mathematical consistency: robust parameter estimation requires PDFs.
- Physical modeling: DSD comes from a random process.
- We want a coherent set of units.
- To build a Z/R relationship with independent parameters.
- To analyze microphysics in terms of physically-meaningful parameters [not a and b].
PDF—based DSD model

How does this approach differs from other DSDs?

\[ n(D) = N_T \cdot p(D) = N_T D^\mu \Lambda^{\mu+1} \frac{e^{-\Lambda D}}{\Gamma(\mu + 1)} \]

Only apparently similar to for instance Ulbrich’s DSD:

\[ n(D)_{Ulbr} = N_0 D^\mu e^{-\Lambda D} \]

[Note that units for \( N_0 \) are \( m^{-4-\mu} \)]
Parameters

\[ N_T, m, \sigma^2 \]

\( m \) and \( \sigma^2 \) are physically related because of hydrodynamics

\[ [m, \sigma^2] \]

are highly correlated in real rainfall

so you end up with just \([N_T, D_m]\)

the largest variation is in \( N_T \)
A Probabilistic View on Raindrop Size Distribution Modeling: A Physical Interpretation of Rain Microphysics

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(Manuscript received 19 February 2013, in final form 20 September 2013)
IN ICE-POP 2018
WE WANT TO DO ANALYSE SUCH DYNAMICS IN 3D AND IN TIME

• Radars
• Models
• Satellites

We are really interested in the dynamics of the Nt
Issue #3

- mp parameterizations are tied to observations
- These observations are seldom comprehensive
- mp may not be good for other regimes / areas
tracing back
‘historical’ assumptions
mp processes
<table>
<thead>
<tr>
<th>Variable</th>
<th>Size distribution</th>
<th>Fall velocity</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_x$ (kg/kg)</td>
<td>$N_x(D)$ (m$^{-4}$)</td>
<td>$U_{dx}$ (m/s)</td>
<td>$\rho_x$ (kg/m$^3$)</td>
</tr>
<tr>
<td>$Q_r$</td>
<td>$N_r(D) = N_{r_0} \exp(-\lambda D)$</td>
<td>$a_r D_r^{b_r} \left( \frac{\rho_0}{\rho} \right)^{1/2}$</td>
<td>$\rho_w = 1 \times 10^3$</td>
</tr>
<tr>
<td>$N_r$</td>
<td>$N_{r_0} = 8 \times 10^6$</td>
<td>$a_r = 842$</td>
<td></td>
</tr>
<tr>
<td>$b_r = 0.8$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q_s$</td>
<td>$N_s(D) = N_{s_0} \exp(-\lambda D)$</td>
<td>$a_s D_s^{b_s} \left( \frac{\rho_0}{\rho} \right)^{1/2}$</td>
<td>$\rho_s = 8.4 \times 10$</td>
</tr>
<tr>
<td>$N_s$</td>
<td>$(N_{s_0} = 1.8 \times 10^6)$</td>
<td>$a_s = 17$</td>
<td>$r_{s_0} = r_0 = 75 \mu m$</td>
</tr>
<tr>
<td>$b_s = 0.5$</td>
<td></td>
<td>$b_s = 0.5$</td>
<td></td>
</tr>
<tr>
<td>$Q_g$</td>
<td>$N_g(D) = N_{g_0} \exp(-\lambda D)$</td>
<td>$a_g D_g^{b_g} \left( \frac{\rho_0}{\rho} \right)^{1/2}$</td>
<td>$\rho_g = 3 \times 10^2$</td>
</tr>
<tr>
<td>$N_g$</td>
<td>$(N_{g_0} = 1.1 \times 10^6)$</td>
<td>$a_g = 124$</td>
<td>$r_{g_0} = r_0 = 75 \mu m$</td>
</tr>
<tr>
<td>$b_g = 0.64$</td>
<td></td>
<td>$b_g = 0.64$</td>
<td></td>
</tr>
<tr>
<td>$Q_c$</td>
<td>mono</td>
<td>$a_c D_c^{b_c}$</td>
<td>$\rho_c = 1.0 \times 10^3$</td>
</tr>
<tr>
<td>$D_i = \left( \frac{6\rho Q_c}{\pi \rho_w N_c} \right)^{1/3}$</td>
<td>$a_c = 3 \times 10^7$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_c = 1 \times 10^8$ m$^{-3}$</td>
<td>$b_c = 2.0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q_i$</td>
<td>mono</td>
<td>$a_i D_i^{b_i} \left( \frac{\rho_0}{\rho} \right)^{0.35}$</td>
<td>$\rho_i = 1.5 \times 10^2$</td>
</tr>
<tr>
<td>$N_i$</td>
<td>$D_i = \left( \frac{6\rho Q_i}{\pi \rho_i N_i} \right)^{1/3}$</td>
<td>$a_i = 7 \times 10^2$</td>
<td>$m_{i_0} = 1 \times 10^{-12}$ kg</td>
</tr>
<tr>
<td>$b_i = 1.0$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(\partial \{N_R, q_R\}/\partial t)_{\text{RacR}}

- Spherical drops
- Fall at terminal \(v\)
- Spaced in the vertical
- Calm, laminar flux
- Large efficiency in collision and coalescence (=1)

**Assumptions**

**Parameterizations**

**Beheng (1994)**

\[(\partial N_R/\partial t)_{\text{accRR}} = -8.0 \cdot 10^3 \cdot N_R \cdot q_R\]
\[(\partial q_R/\partial t)_{\text{accRR}} = 0\]

**Effects on \(N\) and gamma pdf**

\[^\text{V}\] \(^\text{mV} = \) \(^\text{sV} = \)
\[^\text{R}\downarrow \] \(^\text{mR} \uparrow = \) \(^\text{sR} \downarrow = \)
\[^\text{S}\] \(^\text{ms} = \) \(^\text{SS} = \)
\[^\text{I}\] \(^\text{mI} = \) \(^\text{SI} = \)
\[^\text{G}\] \(^\text{mg} = \) \(^\text{SG} = \)

**Pictorial representation**

- Spherical drops
- Fall at terminal \(v\)
- Spaced in the vertical
- Calm, laminar flux
- Large efficiency in collision and coalescence (=1)

**Effects on \(N\) and gamma pdf**
size and shape

4.00 mm

3.68 mm

2.90 mm

2.65 mm

1.75 mm

1.35 mm
\[
(\frac{\partial N_R}{\partial t})_{\text{accRR}} = -8.0 \cdot 10^3 N_R q_R
\]
PSacR $\left( \partial \{N_R, q_R \}/\partial t \right)_{\text{SacR}}$

### Assumptions
- Spherical drops
- Same param b for both psd ()
- Fall at terminal $v$
- Spaced in the vertical
- Calm, laminar flux
- Large efficiency in collision and coalescence ($= 1$)

### Parameterizations
Morrison's mp, based on Ikawa and Saito (1991)

### Effects on $N$ and gamma pdf

<table>
<thead>
<tr>
<th>$N_V$</th>
<th>$m_V$</th>
<th>$S_V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_R \downarrow$</td>
<td>$m_R \uparrow$</td>
<td>$S_R \downarrow$</td>
</tr>
<tr>
<td>$N_S$</td>
<td>$m_S \uparrow$</td>
<td>$S_S \uparrow$</td>
</tr>
<tr>
<td>$N_I$</td>
<td>$m_I = $</td>
<td>$S_I = $</td>
</tr>
<tr>
<td>$N_G$</td>
<td>$m_G = $</td>
<td>$S_G =$</td>
</tr>
</tbody>
</table>
Effects on $N$ and gamma pdf

- $N_V = m_V = S_V =$
- $N_R = m_R \uparrow = S_R \uparrow$
- $N_S \downarrow = m_S \uparrow = S_S \uparrow$
- $N_I = m_I = S_I =$
- $N_G = m_G = S_G =$

Assumptions

- Same param $b$ for both psd (!)
- Fall at terminal $v$
- Spaced in the vertical
- Calm, laminar flux
- Large efficiency in collision and coalescence ($= 1$)

Morrison’s mp, based on Ikawa and Saito (1991)

Parameterizations
Francisco J. Tapiador's precipitation microphysics 2014

Morrison’s mp code

Morrison et al. (2005)

Ikawa&Saito (1991)
c) Collision between rain and snow (see Fig. B-11-2c)

For collision between rain and snow, the accretion rate of rain by snow is

$$
Psacr = \frac{1}{\rho} \int_0^\infty \int_0^\infty \frac{\pi}{4} (Dr + Ds)^2 Er_s |U_{dr} - U_{ds}| \rho_w \frac{\pi}{6} Dr^3 N_{r0} \exp(-\lambda_r Dr) N_{s0} \\
\times \exp(-\lambda_s Ds) dDr dDs.
$$

In most of models so far, the following approximation is used for the differential velocity;

$$
|U_{dr} - U_{ds}| \approx |\overline{U}_r - \overline{U}_s|.
$$

This approximation underestimates Pracs when the value of $\overline{U}_r$ is close to $\overline{U}_s$. To remedy this underestimation, we used the following approximation proposed by Mizuno (1990a) who obtained the exact value of the integral of Eq. (11-55) analytically for the case of $b_r = b_s = 0.5$,

$$
|U_{dr} - U_{ds}| \approx \sqrt{(\alpha \overline{U}_r - \beta \overline{U}_s)^2 + \gamma \overline{U}_r - \overline{U}_s}
$$

with $\alpha = 1.2$, $\beta = 0.95$ and $\gamma = 0.08$. The approximation expressed by Eq. (11-56) yields

$$
Psacr = \pi^2 Er_s \sqrt{(\alpha \overline{U}_r - \beta \overline{U}_s)^2 + \gamma \overline{U}_r \overline{U}_s} \frac{\rho_w}{\rho} N_{r0} N_{s0} \left( \frac{5}{\lambda_r^6 \lambda_s} + \frac{2}{\lambda_r^5 \lambda_s^2} + \frac{0.5}{\lambda_r^4 \lambda_s^3} \right),
$$

The number of collisions between snow and rain particles in unit time is given as

$$
Nsacr = Nracs = \int_0^\infty \int_0^\infty \frac{\pi}{4} (Dr + Ds)^2 Er_s |U_{dr} - U_{ds}| \\
\times N_{r0} \exp(-\lambda_r Dr) N_{s0} \exp(-\lambda_s Ds) dDr dDs.
$$
The Stochastic Collection Equation (SCE)

\[ \frac{\partial N_k}{\partial t} = \frac{1}{2} \sum_{i=1}^{k-1} K_{i,k-i} N_i N_{k-i} - N_k \sum_{i=1}^{\infty} K_{i,k} N_i, \]

is a particular case of

The Smoluchowski Coagulation Equation

\[ \frac{\partial n(x_i, t)}{\partial t} = \frac{1}{2} \sum_{j=1}^{i-1} K(x_i-x_j, x_j) n(x_i-x_j, t) n(x_j, t) - \sum_{j=1}^{\infty} K(x_i, x_j) n(x_i, t) n(x_j, t). \]
which is a particular case of

The Fokker-Planck Equation (FPE)

\[
\frac{\partial W}{\partial t} = \left[ - \sum_{i=1}^{N} \frac{\partial}{\partial x_i} D_i^{(1)}(\{x\}) + \sum_{i,j=1}^{N} \frac{\partial^2}{\partial x_i \partial x_j} D_{ij}^{(2)}(\{x\}) \right] W
\]
Microphysics

Quantitative evolution of the DSD
Microphysics

Quantitative evolution of the DSD

\[
\lim_{t \to s} \frac{1}{t - s} \int_{|y-x| > \varepsilon} p(s, x; t, y) dy = 0;
\]

\[
\lim_{t \to s} \frac{1}{t - s} \int_{|y-x| > \varepsilon} (y - x)p(s, x; t, y) dy = f(s, x) = D^{(1)};
\]

\[
\lim_{t \to s} \frac{1}{t - s} \int_{|y-x| > \varepsilon} (y - x)^2 p(s, x; t, y) dy = g^2(s, x) = D^{(2)};
\]
Microphysics

Quantitative evolution of the DSD

\[
\lim_{t \to s} \frac{1}{t - s} \int_{|y - x| > \epsilon} (y - x) x^{\mu_x} \Lambda^{\mu_x+1} \frac{e^{-\Lambda x x}}{\Gamma(\mu_x + 1)} y^{\mu_y} \Lambda^{\mu_y+1} \frac{e^{-\Lambda y x}}{\Gamma(\mu_y + 1)} dy = D^{(1)}
\]

\[
\lim_{t \to s} \frac{1}{t - s} \int_{|y - x| > \epsilon} (y - x)^2 x^{\mu_x} \Lambda^{\mu_x+1} \frac{e^{-\Lambda x x}}{\Gamma(\mu_x + 1)} y^{\mu_y} \Lambda^{\mu_y+1} \frac{e^{-\Lambda y x}}{\Gamma(\mu_y + 1)} dy = D^{(2)}
\]
Key interests in ICE-POP 2018

· Test parameterizations
· Investigate the dynamics of the DSD and RDSD and check assumptions
· Analyse spatial variability
· Analyse temporal variability
· Advance in the F-P modelling of the DSD evolution: new or newish parameterisation
Thanks

more about UCLM at [www.uclm.es](http://www.uclm.es)

and nice promotional video here:

[https://www.youtube.com/watch?v=wxCRdCnSyPw&feature=youtu.be](https://www.youtube.com/watch?v=wxCRdCnSyPw&feature=youtu.be)